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Lake Dynamics and the Effects of Flooding on Total Phosphorus

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The concentration of total phosphorus in Lake Washington before and after the sewage diversion project was simulated using first-order linear dynamics. Fluctuation in total phosphorus in the lake occurred as a forced response to changes in inflow phosphorus concentration. The dynamics of total phosphorus in Lake Washington was adequately represented by two independent time scales based on water renewal and sedimentation. The water renewal time scale was modelled as a time dependent process. Sedimentation of total phosphorus, on the other hand, appeared constant over the 16-yr period that data were available. A marked increase of total phosphorus in the lake occurred during two flood periods when high concentrations of total phosphorus corresponded to small values of the water renewal time scale (i.e., high flows). At other times, peak concentrations of total phosphorus in the inflow coincided with large values of the water renewal time scale and the lake was dynamically unable to respond to these peaks.

La concentration en phosphore total du lac Washington avant et après la mise en service de la déviation projetée des eaux d'égout, a été simulée selon une dynamique linéaire du premier ordre. Les fluctuations de la concentration en phosphore total dans le lac sont une réponse forcée aux changements de concentration en phosphore dans l'eau d'alimentation. La dynamique du phosphore total dans le lac Washington a été représentée adéquatement grâce à deux échelles chronologiques indépendantes et fondées sur le renouvellement de la masse d'eau et sur la sédimentation. L'échelle de durée de renouvellement de l'eau a été modélisée en fonction du temps. Par ailleurs, la sédimentation du phosphore total paraissait rester constante à l'intérieur de la période de 16-ans pour laquelle les données étaient disponibles. Il s'est produit une augmentation marquée du phosphore total dans le lac à l'occasion de deux crues, alors que les concentrations élevées en phosphore total ont correspondu à des valeurs faibles sur l'échelle chronologique de renouvellement de l'eau (c.-à-d. des débits élevés). À d'autres moments, les concentrations maximales en phosphore total dans l'eau d'alimentation du lac coïncidaient avec les valeurs élevées observées à l'échelle chronologique de renouvellement de l'eau et le lac était incapable, sur le plan dynamique, de répondre à ces pointes.

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Sawyer (1947) observed that when the total phosphorus concentration in a lake was greater than 10 to 20 mg/m^3 algal blooms frequently occurred. The lower limit of Sawyer's observation was used by Vollenweider (1968, 1975, 1976) as the criterion to quantify eutrophication. The problem of predicting the trophic state of a lake is then reduced to one of predicting when the concentration of total phosphorus in a lake, P_L, exceeds 10 mg/m^3 from estimates of phosphorus input to the lake. Towards this end, Vollenweider produced several phosphorus loading models that assume that lakes are in equilibrium with respect to phosphorus. The accuracy of these steady-state models was critically reviewed by Kenney (1982, 1990).

In the present paper the dynamics of total phosphorus in lakes is modelled as a first-order linear system with time dependent coefficients. The model has one source term for phosphorus, the total inflow to the lake. Two sink terms remove phosphorus from the model lake, the outflow from the lake, and sedimentation to the lake bottom. The forced response of the model is calculated using 16 yr of published inflow data from Lake Washington as input. Results are compared with the measured response of Lake Washington following sewage diversion from the lake. Two flood years had a marked effect on the total phosphorus concentrations in Lake Washington.

Lake Phosphorus Dynamics

A Well Mixed Lake with Two Time Scales

Consider a well-mixed lake with a single inflow and a single outflow. Let Q be the volume flow of water into the lake at any time t and assume the lake level is constant with a mean depth z. The surface area of the lake is A and the lake concentration of total phosphorus is P_L. The concentration of phosphorus in the inflowing water is P_i and in the outflowing water is P_L (because the lake is assumed well-mixed). Consider that total phosphorus may also be removed from the lake by sedimentation. Here the assumption is made that the rate of removal by sedimentation is proportional to the concentration of phosphorus in the lake, P_L, and the proportionality constant will be called σ. An alternative suggestion of Edmondson and Lehman (1981) that phosphorus sedimentation in lakes is proportional to phosphorus inflow was refuted by Kenney (1990).

Step Function Response

The step function response describes the increase in lake concentration of phosphorus with time after the sudden start of

phosphorus addition at the inflow. Using similar techniques a sudden decrease can also be calculated. The step function response is important because it permits a ready determination of the time scales or the frequency response characteristics of total phosphorus in a lake.

Here the problem is to determine P_L at any time t after the continuous introduction of phosphorus at the inflow, P_i, beginning suddenly at time $t=0$. The assumption can be made, without loss of generality, that the initial lake concentration was zero at time $t=0$.

If the problem was to be approached from an experimental point of view the first task would be to perform a dimensional analysis of the independent variables, P_i, P_L, Q, A, z, t, and σ. Three dimensionless groups may be found, either by inspection or by using the Buckingham Π theorem (Taylor 1974), as

$$(1)\quad \frac{P_L}{P_i} = \text{Func}\left(\frac{Qt}{Az}, \sigma t\right).$$

Equation (1) states that the dimensionless concentration P_L/P_i is some unknown function of the two dimensionless variables Qt/Az and σt. The actual functional relationship depends upon equations governing the process. Using the empirical approach, the unknown function would be determined by the experiment — by correlating P_L/P_i against Qt/Az and σt, for example. This task could become formidable if the unknown function is nonlinear.

In this simple case, it is not necessary to conduct a series of experiments to determine the unknown function. The equation governing total phosphorus for the model lake can be written directly, using the principle of conservation of mass as,

$$(2)\quad \frac{dP_L}{dt} = \underbrace{\frac{P_i Q}{Az}}_{\text{(source)}} - \underbrace{\frac{Q P_L}{Az}}_{\text{(sink)}} - \underbrace{\sigma P_L}_{\text{(sink)}}$$

This equation has one source term and two sink terms. The rate of change of phosphorus in the lake, dP_L/dt, is equal to the difference between the rate at which the concentration is increasing because of phosphorus in the inflow and the rate at which the concentration is decreasing because of losses to the outflow and sedimentation.

Equation (2) is a first order ordinary differential equation that can be readily solved by separation of variables. The solution, subject to the initial conditions, $P_L=0$ at $t=0$, is:

$$(3)\quad \frac{P_L}{P_i} = \frac{1}{1+\dfrac{Az\sigma}{Q}}\left(1 - \exp(-Qt/Az)\exp(-\sigma t)\right).$$

Thus, the unknown function in equation (1) has been found for the 'model' lake. If the governing equation was different due to a different set of initial assumptions (say, about mixing), then this unknown function would also be different.

The form of the solution (i.e. the step function response in equation (3)) is shown in Fig. 1 for several different lakes. In each case, the concentration of total phosphorus in the lake increases rapidly following the step increase in the inflow concentration at time zero and eventually reaches an asymptotic value at steady-state.

The term Az/Q is one time scale, τ_w, controlling the dynamics of total phosphorus in lakes. It is known as the water renewal time or water residence time. The second time scale, $\tau_s = 1/\sigma$, will be called the sedimentation time scale. Because of the form

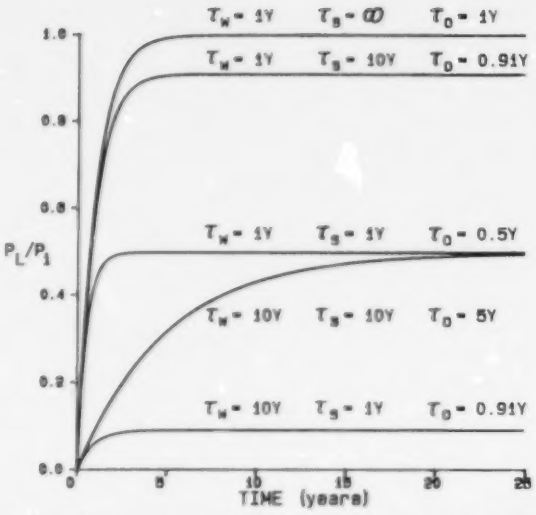

FIG. 1. Step function response of total phosphorus in lakes with two time scales: τ_w, the water renewal time scale and τ_s, the sedimentation time scale. The overall time scale, τ_0, is also given for each lake. Note that a time of $5\tau_0$ is required to reach 99% of the steady-state value.

of the solution for this model an overall time scale, τ_0, can also be determined as,

$$(4)\quad \tau_0 = \frac{1}{\dfrac{1}{\tau_w}+\dfrac{1}{\tau_s}} = \frac{\tau_w}{1+\dfrac{\tau_w}{\tau_s}}$$

The overall time scale for lake phosphorus is always less than the water renewal time for any finite value of sedimentation.

In the steady-state,

$$(5)\quad \frac{P_L}{P_i} = \frac{1}{1+\dfrac{Az\sigma}{Q}} = \frac{1}{1+\dfrac{\tau_w}{\tau_s}} \cdot (t \to \infty)$$

At steady-state, the total phosphorus concentration in a lake with no sedimentation is equal to the inflow concentration (i.e. $P_L = P_i$ for $\tau_s \to \infty$). To reach a reasonable estimate of the final steady-state value (say 99%) requires a time of $t = 5\tau_w$ (or $5\tau_0$). For all values of $t < 5\tau_w$, $P_L < P_i$.

With sedimentation, the steady-state lake concentration is always less than the inflow concentration. The steady-state value of P_L depends not only on water renewal but also on the sedimentation time scale. Therefore, two vastly different lakes (one with a high flushing rate and a high sedimentation rate and the second with low flushing and low sedimentation) can have the same steady-state value of lake concentration (Fig. 1). The time required to reach steady-state, however, is much longer in the latter case. In general, the higher the flushing rate (i.e. the smaller τ_w) for a given sedimentation rate, the closer the steady-state value of P_L will be to P_i. Conversely, the higher the sedimentation (i.e. the smaller τ_s) for a given flushing rate, the smaller the final value of P_L. As before, 99% of the steady-state value is reached in a time equal to $5\tau_0$.

Forced Response

Monitoring lake concentrations after a sudden change in input is a useful method to determine the dynamics of lake phospho-

rus. For very small lakes, this approach is possible empirically by conducting 'whole lake experiments.' For most lakes, however, experimental manipulation is not practical. In general, the phosphorus input is neither constant nor subject to sudden changes. Rather, it is continuously changing. The lake concentration is also constantly changing in an attempt to follow the input concentration. How closely the lake concentration follows the changes in inflow concentration (or forcing) depends upon the lake phosphorus dynamics (i.e. τ_w,τ_s, or τ_σ). For forcing frequencies that are small compared with τ_σ^{-1}, the lake concentration follows the forcing exactly. For forcing frequencies that are high compared with τ_σ^{-1}, the lake concentration shows little or no response to changing inflow concentrations.

Equation (2) has been rewritten to explicitly show the time dependent forcing by P_i as,

$$(6)\quad \tau_w(t)\frac{dP_L(t)}{dt} + \{1 + \sigma(t)\tau_w(t)\}P_L(t) = P_i(t).$$

Equation (6) also has time dependent coefficients for both τ_w and σ. It was implicitly assumed earlier that these coefficients were constants. Most temperate lakes, however, are subjected to large variations in flow (Q) throughout the year, with peak flows usually occurring during spring runoff. Changes in lake level (z) and volume (Az) may also be large, particularly for lakes used for hydroelectric purposes. These variables directly affect the water renewal time scale, making τ_w time dependent.

The factors that affect σ are not as well understood. It is known that physical sedimentation is a nonstationary random process that can be positive or negative at various times. A negative sedimentation flux (i.e. resuspension) occurs to some extent in the water column of all lakes and is very common in shallow lakes. Because physical sedimentation is controlled principally by the level of turbulence in a lake seasonal variations in σ are possible, reflecting differences between turbulence under winter ice and during open water in temperate lakes. Biological processes may also play an important role in sedimentation of total phosphorus by controlling the rate of particle generation in the water column.

Temporal variations in the coefficients may be more important than variations in the phosphorus concentration of the inflow (forcing) for many lakes. In very complicated lakes, the forcing and the time dependent coefficients may be coupled; say, by the flow through the lake, Q. Before discussion of a specific numerical example, it may be stated in general that a large spring runoff tends to make the spring lake phosphorus concentration equal to the spring inflow concentration by greatly reducing the instantaneous value of τ_w (and τ_σ).

Before equation (6) can be solved, the temporal variation in τ_w and σ must be known (or independently 'predicted') either in the form of time histories for random variations or in the form of analytical functions for periodic variations. The model forcing must also be input to the model. For a hindcast, $P_i(t)$ is usually input as time histories of measured phosphorus inflow to a lake. Anticipated changes in phosphorus inflow, however, may also be input to the model (either as a time series or as an analytical function) in order to forecast future lake changes.

Phosphorus Dynamics in Lake Washington

Input Data

Edmondson and Lehman (1981) published an excellent set of data tracing the recovery of Lake Washington following diversion of sewage from the lake. These data represent as close an approximation to step function forcing as one could expect for such a large lake. Annual mean values of total phosphorus in the inflow to the lake were presented for the period 1962 to 1978 and monthly mean data were given from 1970 to 1978. Instantaneous values of the concentration of total phosphorus in the lake were also presented as determined on January 1 each year from 1962 to 1978. The lake concentration data were spatial averages from an unspecified number of locations.

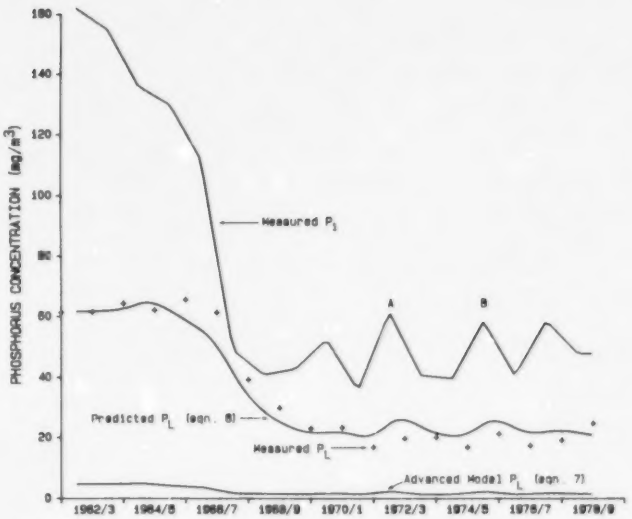

FIG. 2. Simulated responses of P_L, the concentration of total phosphorus in Lake Washington, produced by P_i, the annual mean values of the phosphorus inflow measured from 1962 to 1978. The instantaneous lake concentration of total phosphorus measured on January 1 of each year are also shown.

Total phosphorus concentrations in Lake Washington were simulated using equation (6) as the phosphorus dynamics model. Integration was performed with a variable step, fourth order, Runge-Kutta routine with a base time of 1 d.

Daily inputs to the model $P_i(t)$ and $\tau_w(t)$ were linearly interpolated from the published data. No sedimentation data were given in Edmondson and Lehman (1981) that would permit one to determine the sedimentation time scale as a function of time. A value for τ_s was calculated using equation (5) assuming that Lake Washington was approximately in equilibrium at the beginning of the available data set. In effect, τ_s was estimated by a one-point calibration using conditions existing in 1962 and assumed thereafter to be a constant.

The model output was the instantaneous lake phosphorus concentration at noon each day.

Simulated Lake Phosphorus Concentrations

Figure 2 shows the rapid drop in $P_i(t)$ following sewage diversion, the lake response $P_L(t)$ as calculated from the annual mean inflow concentration from 1962 to 1978 and the measured values of lake phosphorus. The predicted lake phosphorus matches the measured values very well considering that the input data were interpolated from annual averages and both the measured and the predicted lake concentration were instantaneous values. Note, in particular, that the model P_L represents the measured P_L well both before and after diversion, in addition to predicting the exponential recovery phase. The match between predicted and measured lake phosphorus before the

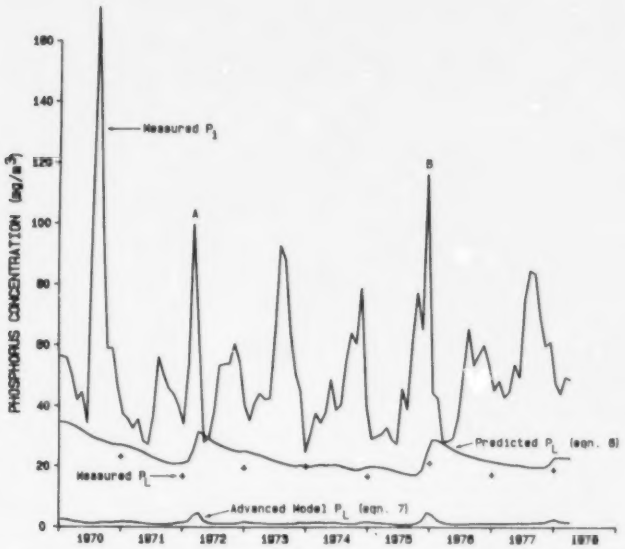

FIG. 3. Simulated responses of P_L, the concentration of total phosphorus in Lake Washington, produced by P_i, the monthly mean values of the phosphorus inflow measured from 1970 to 1978. The instantaneous lake concentration of total phosphorus measured on January 1 of each year are also shown.

diversion is simply a reflection of the one-point model calibration for 1962. To accurately simulate both the exponential decay and the final asymptote, however, requires both time scales to be correct. This dynamic constraint represents a stringent test of the model (see equations (3) and (5) and the earlier discussion of Fig. 1).

Figure 3 shows additional model results using the measured monthly mean data from 1970 to 1978. The monthly forcing P_i in Fig. 3 is substantially more variable than the annual forcing shown in Fig. 2. In spite of increased fluctuations in the inflow phosphorus, the instantaneous lake phosphorus concentrations predicted by the model are similar in both figures. In fact, P_L in Fig. 3 matches the measured values better than the P_L in Fig. 2. Because the model can only simulate a response to the data input, the more closely the input represents the actual forcing of the lake the better is the expected result. Presumably, weekly (or even daily) input data would produce an even better simulation.

Steady-State Models

For comparison, P_L was also simulated using the results of a steady-state phosphorus loading model (Vollenweider 1968, 1975, 1976). The concept of phosphorus loading to a lake, $L = P_i\ Q/A$, was used by Vollenweider when evaluating lake response to elevated inputs of phosphorus from municipal and industrial sources. Because the concentration of phosphorus in a lake clearly scales with the inflow concentration and not with the loading to the lake (see equation (3)), the phosphorus loading concept is of little value in a dynamic framework. Instead, the results of Vollenweider's steady-state models are applied here using the fundamental variables given in equation (1).

The conservation of mass equation is the basic starting point for both steady-state and dynamic models. For example, equation (2) is mathematically equivalent to the nonconservative phosphorus loading model presented by Vollenweider (1975, p. 58). To distinguish it from Vollenweider's earlier models, we will refer to it as the advanced model.

The advanced model evolved from equation (2) by incorporating the results of a regression analysis between P_L/P_i and τ_w using spring phosphorus data from 21 lakes and annual mean values of τ_w (Vollenweider 1976). The data were scattered about a regression line that was strongly influenced by two lakes with large values of τ_w. Vollenweider did not use the actual regression equation in his steady-state model but rather an analytical expression that better represented the asymptotic behaviour of P_L/P_i near $\tau_w = 0$. A relationship between the two time scales (i.e. $\tau_s = \sqrt{\tau_w}$) follows directly from this analytical equation.

Vollenweider only applied this relationship in his steady-state advanced model. It is possible, however to use Vollenweider's result for a dynamic simulation of P_L by substituting the relationship into the differential equation governing phosphorus for the advanced model. Then equation (2) becomes,

$$(7)\quad \frac{dP_L}{dt} = \underbrace{\frac{P_i Q}{Az}}_{\text{(source)}} - \underbrace{\frac{P_L}{\tau_w}}_{\text{(sink)}} - \underbrace{\frac{P_L}{\sqrt{\tau_w}}}_{\text{(sink)}}.$$

Note that, although equation (7) contains one source term and two sink terms, the two sink terms are not independent. The sedimentation sink is now a function of water renewal. Rather than a model based on two independent time scales (τ_w, τ_s), the advanced model depends solely on the water renewal time, τ_w.

A strong advantage of equation (7) is that no calibration is required to simulate to the dynamics of total phosphorus in any lake. If it works for one lake, it should work for all lakes. Furthermore, only two inputs, $P_i(t)$ and $\tau_w(t)$, are needed to predict P_L and both inputs can be readily measured.

Equation (7) was applied to the Lake Washington data without explicit consideration of the confidence limits associated with Vollenweider's regression analysis. Unfortunately, P_L obtained from equation (7) does not simulate the measured lake concentrations very well (Figs. 2 and 3). The values of P_L calculated from Vollenweider's advanced model are an order of magnitude less than the P_L measured in Lake Washington. These small values of concentration indicate that the sedimentation sink is far too dominant in Equation (7).

The reason for the failure of the advanced model when applied to a dynamic simulation of Lake Washington is rooted in Vollenweider's regression analysis. All of the phosphorus loading models described in Vollenweider (1975, 1976) make use of the assumption that the 21 lakes were in a steady state with respect to phosphorus. If this assumption were true equation (7) should accurately model the dynamics of total phosphorus, at least, in these 21 lakes. Unfortunately, the potential margin for error when ignoring the time dependent term in the differential equation is very great. When the steady state assumption is made incorrectly (i.e. by assuming lakes are in equilibrium when they are not), the error is simply rolled up and included in the calculation of the sedimentation term. The most likely result is an over-estimation of the effects of sedimentation. While it is outside the scope of the present paper, a detailed assessment of the impact of the steady state assumption on Vollenweider's advanced model may be found in Kenney (1990)

Effect of Flooding on Total Phosphorus in Lake Washington

Comparing the fluctuations in P_i with the lake response in Figs. 2 and 3 clearly shows a fundamental characteristic of the

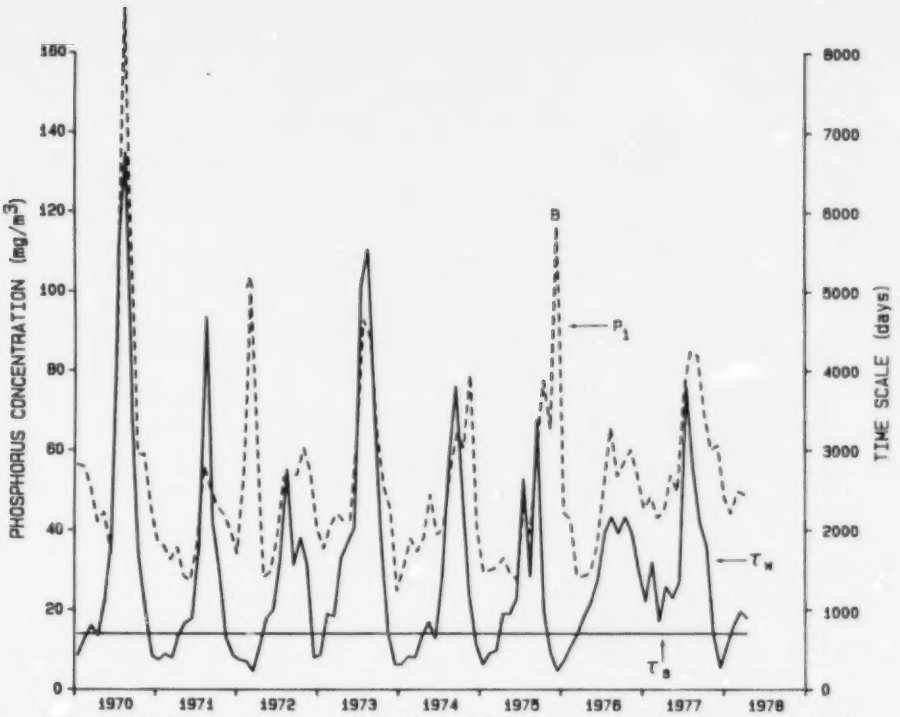

FIG. 4. Time scales input to phosphorus dynamics model (equation (6)) for the 1970 to 1978 simulation. The water renewal time scale, τ_w, was calculated from the measured monthly discharge. The sedimentation time scale, τ_s, was assumed constant. The forcing $P_i(t)$ for the model was the monthly mean concentration of total phosphorus input to Lake Washington. Daily values required for input to the model were linearly interpolated from the monthly data.

dynamics of total phosphorus in lakes. Forcing frequencies that are too high for the lake to respond are eliminated from the output while low frequencies are closely followed by the lake with little or no attenuation.

In both Figs. 2 and 3, the simulated P_L also has two significant peaks, one in the spring of 1972 and a second in the fall of 1975. These P_L peaks appear to be a response to the peaks in inflow phosphorus marked A and B. The question arises, therefore, why does the lake respond to the inflow peaks denoted A and B but not to other more intense peaks?

The answer may be found in Fig. 4, which shows the time scales which define the phosphorus dynamics of the model, τ_w and $\tau_s = 1/\sigma$, in addition to P_i. Note in Fig. 4 that most peaks in inflow phosphorus occurred during periods of low discharge (i.e. high values of τ_w). An inverse relationship between discharge and inflow concentration is typical of many lakes because large runoff tends to dilute chemical constituents in the inflow, particularly if the runoff is melt water. These P_i peaks were entirely damped and had no effect on the lake phosphorus because the overall time scale of the lake at the time of each peak was too long to permit the lake to respond. The two P_i peaks marked A and B did produce corresponding peaks in P_L, however, because the lake was dynamically able to respond at the time they occurred (i.e. τ_w (and τ_o) was small when P_i was large).

The A and B peaks resulted from the only two floods that occurred in Lake Washington's watershed from 1962 to 1978. During floods, high values of P_i coincide with high discharge because rivers overflow their banks and increase erosion of total phosphorus from soils not normally submersed. Edmondson and Lehman (1981) noted that the increase in P_i during the two floods was due to an increase in particulate phosphorus (P) with the dissolved P remaining relatively constant. Although the two floods produced significant lake response in the simulated P_L using both annual and monthly mean input data, the once a year temporal resolution of the P_L measured by Edmondson and Lehman is not high enough to verify the occurrence of these peaks in the lake.

Discussion

My model results suggest that two independent time scales, water renewal and sedimentation, are sufficient to represent the phosphorus dynamics of Lake Washington. Moreover, the success of a constant value for the sedimentation time scale suggests that there is no long term trend in the rate of phosphorus sedimentation. This was expected because both physical and biological processes controlling sedimentation are likely to be stationary over the long term. Because Lake Washington does not freeze over in winter, the physical processes that control sedimentation may also be relatively constant throughout the year. Measurements of sedimentation flux presented in Edmondson and Lehman (1981), however, suggest that sedimentation may be correlated with algal bloom events on a time scale less than 1 yr. Unfortunately, sufficient data are not available to resolve the question of short-term time dependence of the sedimentation time scale. Certainly, a constant value of τ_s allows a reasonably accurate simulation of both the exponential recovery phase and the asymptotic concentration of total phosphorus in Lake Washington. On the other hand, a dynamic sim-

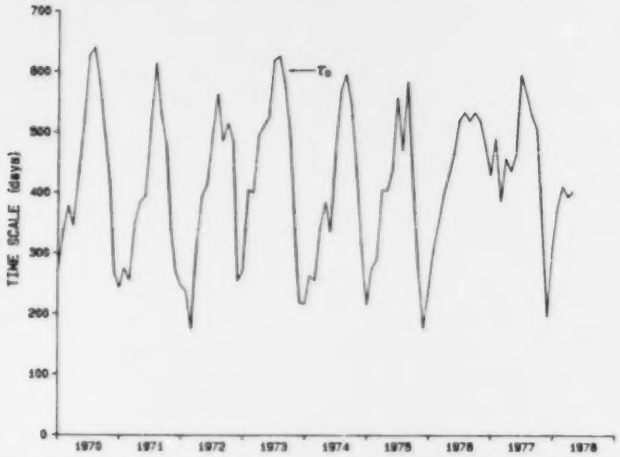

FIG. 5. Overall time scale, τ_0, controlling the dynamics of total phosphorus in Lake Washington.

ulation based on Vollenweider's advanced model with $\tau_s = \sqrt{\tau_w}$ underestimated the concentration of total phosphorus in Lake Washington by an order of magnitude.

It is premature to extrapolate these results too far because the Lake Washington data are unique, not only in the step function change in inflow phosphorus but also in the quality of the data and the length of the record. The rather small value of the sedimentation time scale (706 d) obtained for Lake Washington, however, is noteworthy. If other large lakes with physical characteristics similar to Lake Washington have similar sedimentation time scales then sedimentation would dominate the overall lake response regardless of very long water renewal times. The overall time scale of total phosphorus, τ_0, clearly shows the influence of sedimentation in Lake Washington (Fig. 5). During the summer months when the water renewal time scale becomes very large, the overall time scale approaches an upper limit equal to the sedimentation time scale. This means, for example, that a lake with a very long water renewal time, like Lake Superior or Lake Tahoe, might respond to phosphorus removal schemes at about the same rate as Lake Washington. While some physical arguments can be advanced in support of similarity in sedimentation in very large lakes, hard evidence is lacking. On the other hand, lakes shallow enough to be subject to sediment resuspension are expected to be very different. Shallow lakes are likely to require a time dependent τ_s for successful modelling.

In conclusion, total phosphorus concentrations in Lake Washington were successfully modelled over a 16-yr period using first-order linear dynamics. Two independent time scales, based on water renewal and sedimentation, are adequate to define the lake phosphorus dynamics. The water renewal time scale was highly variable throughout the year. On the other hand, the sedimentation time scale was successfully modelled as a constant with a value of 706 d. Further work is required to determine if sedimentation behaves similarly in other lakes.

Two floods that occurred during the 16-yr period had a marked effect on the simulated total phosphorus in the lake. These flood periods were the only occasions when high concentrations of total phosphorus in the inflow coincided with high flows. All other peaks in inflow phosphorus had little effect on the lake concentration because they occurred at times when the lake was dynamically unable to respond.

Acknowledgements

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